Pre-class Warm-up!!!

How would you do the following question?

Spring 2007 Midterm 3
2. Compute the integral ∬_S F · dS , where F(x,y,z) = (y-z, x-z, x-y) and S is the planar surface parametrized by Phi (u,v) = (u-v, u+v, u) for $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Orient the surface so the first component of the normal vector is positive.

a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem

e. None of the above.

Review for exam 3

' We have covered

$\nabla \cdot F = 0$ $\nabla \times F = 0$

8.3 Conservative vector field (criterion for F = grad f, also for F = curl G. All past exam questions ask to find f so that F = grad f and then evaluate a line integral $\int F \cdot dS$, which was done in section 7.2 and on Exam 2.) There is no question like this on Exam 3

$= f(c(b)) - f(c(a))$

6.1, 6.2 Change of variables in 2D and 3D. (Finding linear changes of variables, the change of variables formula, cylindrical and spherical coordinates) $\iint_D f\, dx\, dy = \iint_{D^*} f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv$

7.3 What is a parametrization of a surface? (Find tangent planes) tangent vectors T_u, T_v. find
E

7.4 Surface area. $\iint_D \|T_u \times T_v\|\, du\, dv$

7.5 Scalar surface integrals $\iint f \|T_u \times T_v\|\, du\, dv$

7.6 Vector surface integrals $\iint F \cdot d\underline{S}$

8.2 Stokes' theorem

$$\iint_{\partial S} F \cdot d\underline{s} = \iint_S \nabla \times F \cdot dS$$

Things not on Exam 3

Find f so that F = grad f, then use this to compute a line integral.
This was material from section 7.2 and appeared in Exam 2.

Green's theorem.

cylindrical (r, θ, z) $dx\, dy\, dz = r\, dr\, d\theta\, dz$

spherical (ρ, θ, ϕ). formula given in the exam.

The past Spring exams 2007 - 2011

All have a question about Stokes′ theorem. Three do Stokes on a triangle, one does it on a hemisphere, and one on a disk parallel to the xy-plane.

Typically they have one or two questions about cylindrical or spherical coordinates.

They all have a change of variables question.

The mostly have a gradient vector field question. Not on our exam.

Spring 2007 Midterm 3

2. Compute the integral ∬_S F · dS , where F(x,y,z) = (y-z, x-z, x-y) and S is the planar surface parametrized by Phi (u,v) = (u-v, u+v, u) for $0 \le u \le 1$ and $0 \le v \le 1$. Orient the surface so the first component of the normal vector is positive.

Solution 1st approach: Try Stokes' theorem

Notice that $F = \nabla \times G$ for some G because

$$\nabla \cdot F = \frac{\partial}{\partial x}(y-z) + \frac{\partial}{\partial y}(x-z) + \frac{\partial}{\partial z}(x-y) = 0$$

In fact $G = \left(\frac{y^2 - z^2}{2}, \quad , \quad \right)$

so $\iint_S F \cdot dS = \int_{\partial S} G \cdot ds.$

∂S has 4 pieces (parallelogram). It's a lot of work!

2nd approach. $T_u = (1,1,1)$, $T_v = (-1,1,0)$

$T_u \times T_v = (-1,-1,2)$. This Φ is not consistent with the orientation.

We could do instead $\Phi_1(v,u) = \Phi(u,v)$

We get $T_v \times T_u = (1,1,-2)$.

Or Use Φ and multiply by -1 at the end.

Use Φ_1.

$$\iint_S F \cdot dS = \int_0^1\int_0^1 (u+v-u, u-v-u, (u-v)-(u+v)) \cdot (1,1,-2)\, du\, dv$$

$$= \int_0^1\int_0^1 (v, -v, -2v) \cdot (1,1,-2)\, du\, dv$$

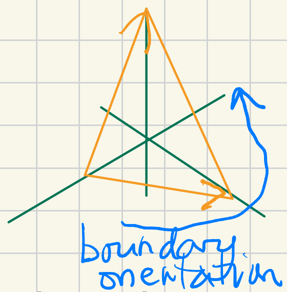
$$= \int_0^1\int_0^1 4v \, du\, dv = 2$$

Spring 2010 Question 6.
Evaluate

$\int_C (x+y)dx + (2x-z)dy + (y+z)dz$

where C is the perimeter of the triangle connecting (2,0,0), (0,3,0) and (0,0,6) in that order.

How would you do it?



a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem ✓

e. None of the above.

Solution Use Stokes: $\int_C F\cdot ds$

$$= \iint_{\substack{\text{triangle with}\\ \text{those vertices, suitably oriented.}}} \nabla\times F\cdot dS$$

$F = (x+y, 2x-z, y+z)$

$\nabla\times F = (1-(-1), 0-0, 2-1) = (2,0,1)$.

Parametrize the triangle: $(0,3,0)-(2,0,0)$, $(0,0,6)-(2,0,0)$

$\Phi(u,v) = (2,0,0)+u(-2,3,0)+v(-2,0,6)$

$T_u = (2,0,-6)$ $T_v = (-2,3,0)$

$T_u\times T_v = (18,12,6)$. This has correct orientation. Let $\underline{n} = \frac{T_u\times T_v}{\|T_u\times T_v\|}$ be a unit normal.

$$\iint_S \nabla\times F\cdot dS = \iint_D \nabla\times F\cdot \underline{n}\,\|T_u\times T_v\|\,du\,dv$$

$$= \frac{42}{\|T_u\times T_v\|}\iint_D \|T_u\times T_v\|\,du\,dv = \frac{42}{\|T_u\times T_v\|}\cdot \text{Area of } S$$

$$= \frac{42}{\|T_u\times T_v\|}\cdot\frac{1}{2}\cdot\|T_u\times T_v\| = 21$$

Spring 2010 Question 1. Parametrize the surface $3(x^2+y^2) + 2z^2 = 2$ with $z \geq x^2+y^2$.

How would you do it?

a. Evaluate directly

b. Go into polar, cylindrical or spherical coordinates.

c. Make a different change of variables.

d. Use Stokes' theorem

e. None of the above.

Spring 2008 Question 4. Parametrize the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. Include the correct bounds

Find an equation for the tangent plane when theta = phi = $\pi/4$.

Spring 2010 Question 5. Let Be be the region in the first quadrant bounded by the curves $xy = 1$, $xy = 3$, $x^2-y^2 = 1$ and $x^2-y^2 = 4$.

Evaluate $\iint_B (x^2+y^2)dx\,dy$

Using the change of variables $u = x^2 - y^2$, $v = xy$.